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Production Smoothing When Bank Loan Supply Shifts: The Role of Variable Capacity Utilization

How do firms smooth production when facing financing uncertainty? By using a model incorporating financing constraints, this paper shows that firms may adjust capacity utilization rates to buffer against financing disturbances. In particular, it emphasizes that variable capacity utilization plays the roles of both inter- and intratemporal substitution of capital in this context. The paper presents results from the comparative statics and numerical calibrations of the model. These results show that the implied short-run dynamics are consistent with business cycle phenomena. The results also indicate that the long-run average of the capital stock is not likely to be affected by financing uncertainty, so that stabilization policy in the banking sector may have only a second-order welfare gain.

WHAT ARE THE IMPLICATIONS for a firm's investment and production behavior when financing constraints are anticipated? If investment is constrained by external finance, does the level of capital stock change one-for-one as implied by the textbook capital accumulation equation? Is the impact on the capital stock the same in the short run as in the long run?

While important in their own right, the answers to the above questions are also of interest in relation to other theories and hypotheses. For instance, the lending view of monetary policy (see, for example, Bernanke 1993) asserts that monetary policy has a significant impact on bank loan supply, and that shifts in the supply of bank loans have a direct and nontrivial impact on output. While there are many studies that consider the first aspect of the lending view, studies related to the second aspect are few and far between. The results presented in this paper should make up for this lack.

This paper presents a model in which the steady state is characterized by the presence of financing constraints and excess (that is, less than 100 percent) capacity. Financing constraints in the steady state take the form of a markup of borrowing cost over the opportunity cost of funds. This is broadly consistent with observations from

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the U.S. financial market where the lending rate is usually the prime rate plus a markup. The excess capacity arises due to the user cost of wear and tear. It then shows that, with variable capacity utilization rates, firms can smooth out variations in stocks and outputs in an environment characterized by financing uncertainty. In particular, it is shown that capacity utilization performs both inter- and intratemporal substitutions of capital. In times when lower stocks are foreseen, firms decrease their capacity utilization to conserve capital. When the adverse effect on stocks materializes, firms increase their utilization rates to smooth the flow of capital services.

Both short-run and long-run implications of the model are studied. In the case of the short run, the implied cyclical behavior is consistent with some of the observed business cycle phenomena. As for the long run, it is shown that financing uncertainty is not likely to have a first-order effect on the distribution of the capital stock. The implication to the lending view hypothesis of monetary policy is that fluctuations in bank loan supply may have short-run effects on output, but the long-run effect is likely to be neutral. This long-run neutrality draws support from Driscoll (1994), whose study focuses on twenty-seven years of U.S. annual panel data and who finds that while shocks to money demand have a significant impact on output, shocks to bank loan supply do not have any significant impact.

The model also provides a plausible channel through which firms buffer output against financing constraints. A conventional explanation of output smoothing in the event of changes in bank loan supply resorts to the possibility that firms substitute for other forms of finances. Gertler and Gilchrist (1994), however, argue that only very large firms have such substitution ability. On the other hand, the mechanism whereby firms adjust capacity utilization is viable for all kinds of firms, regardless of their asset sizes or market power. Furthermore, the mechanism is applicable to shifts in all types of external finance, not necessarily restrict to bank credit.

The part of production smoothing in this paper bears a resemblance to the literature of consumption and buffer-stock savings. Deaton (1991) and Carroll (1997) show that borrowing constraints with precautionary saving motives cause consumers to accumulate assets when times are good in order to protect themselves when times are bad. Other evidences of buffer-stock savings are provided in a number of papers including Dardanoni (1991) and Carroll and Samwick (1997, 1998). The similarity between consumption and production smoothing is apparent: agents build up stocks to smooth out current and future stock consumption when constraints in consuming the stocks are anticipated. In the consumption literature, this requires "prudent" behavior of consumers, which imposes certain conditions on utility functions (Kimball 1990). In the current paper, production smoothing relies on the mechanism whereby capacity utilization responds to the shadow cost of capital.

The remainder of the paper is organized as follows. Section 1 sets up the model. In section 2, a comparative static analysis is carried out. The inter- and intratemporal substitution of capital is discussed here, as well as the model's ability to generate some stylized facts of business cycles. Section 3 solves the dynamic programming problem of the model; the results are used to do impulse response analysis and examine the long-run effect on capital accumulation. Section 4 concludes the paper.

1. THE MODEL

This section presents a model of investment with financing constraints and variable capacity utilization. The modeling of variable utilization follows Greenwood, Hercowitz, and Huffman (1988). Assume that a representative firm uses capital stock (K) and labor stock (L) as factor inputs. Output (Y) is a function of the inputs and the capacity utilization rate (β):

$$Y_t = F(\beta_t K_t, L_t) .$$

The capacity utilization rate together with the capital stock determine the flow of capital services used in production. For simplicity, β_t is not modeled as a function of the labor input. This can be rationalized by either assuming that β_t depends on the "good will" of labor which could be independent of the labor stock, or following Greenwood, Hercowitz, and Huffman (1988) that while L_t represents the total labor employed, β_t reflects the portion used directly in production, with the remainder being involved in maintenance activities. The function F is assumed to satisfy the concavity conditions needed for a maximization problem: $F_1 > 0$, $F_2 > 0$, $F_{11} < 0$, $F_{22} < 0$, $F_{12} > 0$, and $F_{11}F_{22} - F_{12}^2 \geq 0$.

Factor inputs are assumed to be quasi-fixed, and the transition equations for the capital and labor stocks are

$$K_{t+1} = (1 - \delta(\beta_t))K_t + i_t ,$$

$$L_{t+1} = (1 - d)L_t + l_t ,$$

where δ is the capital depreciation rate and is a function of the utilization rate, and d is the quit rate of labor. The function δ implies that there is a pure user cost associated with the rate of capacity utilization (Johnson 1994). Assume $\delta' > 0$ and $\delta'' > 0$ so that greater utilization implies more wear and tear, and perhaps less maintenance.

There are fixed costs associated with each unit of newly installed capital (i) and newly hired labor (l). Without loss of generality, we normalize the cost of installing a unit of capital to one, and the cost of recruiting labor, which includes advertising, training, and making uniforms, etc., equals a. Therefore in order to install i units of capital and hire l units of labor, a firm has to invest $I = i + al$ dollars. Although theoretically a firm could finance I from retained earnings or borrowing, we focus on the latter by assuming that the firm's profit is totally distributed to shareholders, so that it has to borrow from the credit sector in order to invest.[1]

For each dollar the firm has borrowed at the beginning of period t, it has to pay back $C(I_t, \theta_t)$ at the end of the period, where function C is assumed to be twice differentiable in both of its arguments and θ_t is a stochastic variable relating to financing shocks. Assume $C_1 \geq 0$, $C_2 \leq 0$, and $C_{12} \leq 0$, so that financing constraints are

1. This is a partial equilibrium model, and does not model the credit sector explicitly.

represented by decreases in θ_t. Assume also that θ_t has a stationary Markov distribution function $\Psi(\theta_t|\theta_{t-1})$. As will be shown in section 3, θ_t can assume different roles in a cost function: a shifter of the intercept, a threshold quantity representing rationed credit, and a variable affecting the shape (that is, the second derivative) of the cost function. All these can be modeled under a unified framework with different parameter values, and therefore different types of financing constraints can each be thought of as being a close variant of another.

Finally, assume the product market is competitive without uncertainty so that $P_t = Z$ is a constant price parameter. The firm's wage bill is simply WL_t. The firm chooses β_t, i_t, and l_t to maximize profit:

$$\max_{\{\beta_t, I_t, l_t\}} E\sum_{j=0}^{\infty} \Delta^t \{ZF(\beta_{t+j}K_{t+j}, L_{t+j}) - C(I_{t+j}, \theta_{t+j})I_t - WL_{t+j}\} \tag{1}$$

subject to

$$K_{t+1} = (1 - \delta(\beta_t))K_t + i_t , \tag{2}$$

$$L_{t+1} = (1 - d)L_t + l_t , \tag{3}$$

$$I_t = i_t + al_t ,$$

where E is an expectations operator, and Δ is a time-discount factor.

2. COMPARATIVE STATICS

2.1 The Inter- and Intratemporal Substitution of the Capital Stock

To simplify the algebra, we focus on the special case where $C(I_t, \theta_t) = C(0, \theta_t)$, so that $C_1 = C_{12} = C_{21} = 0$. This implies that the unit cost does not depend on the total amount borrowed, and that financing shocks shift the cost schedule by changing the cost function's intercept. The more general cost function will be resumed in section 3, where it is also shown that many of the results presented here are preserved in the more general case. An auxiliary condition following from this assumption on the cost function is that $F_{11}F_{22} - F_{12}^2$ must be greater than, but not equal to, zero in order to satisfy the second-order condition of the corresponding maximization problem. For the purpose of comparative statics, we further assume that θ_t is not serially correlated, thus $\Psi(\theta_t|\theta_{t-1}) = \Psi(\theta_t)$. Again, the more general assumption of θ_t will be resumed in section 3.

The firm's optimization problem of (1) is, thus,

$$V(K_t, L_t, \theta_t) = \max_{\beta_t, K_{t+1}, L_{t+1}} \{ZF(\beta_t K_t, L_t) - C(I_t, \theta_t)I_t - WL_t + \Delta EV(K_{t+1}, L_{t+1}, \theta_{t+1})\}$$

subject to

$$I_t = i_t + al_t\,,$$

$$i_t = K_{t+1} - (1 - \delta(\beta_t))K_t\,,$$

$$l_t = L_{t+1} - (1 - d)L_t\,.$$

To keep the notations uncluttered, we shall use the superscript "+" to denote period $t + 1$ variables, and subscripts for partial derivatives. Similarly, a function with a "+" superscript denotes a function having period $t + 1$ arguments, on the basis of the understanding that all function forms in the model are time invariant. The first-order conditions of β, K^+, and L^+ are, respectively,

$$0 = ZF_1(\beta K, L) - C(I, \theta)\delta'(\beta)\,, \tag{4}$$

$$0 = -C(I, \theta) + \Delta E\{ZF_1(\beta^+K^+, L^+)\beta^+ + C(I^+, \theta^+)(1 - \delta(\beta^+))\}\,, \tag{5}$$

$$0 = -aC(I, \theta) + \Delta E\{ZF_2(\beta^+K^+, L^+) + C(I^+, \theta^+)[a(1 - d^+)] - W\}\,, \tag{6}$$

where β, K^+, and L^+ are all functions of θ, K, and L.

Equation (4) implies that the marginal revenue of β should equal the replenishment cost of capital due to the higher rate of wear and tear. Equation (5) implicitly equates the current cost with the expected revenue from installing an additional unit of capital. The cost of installing additional capital in period t is simply C, and in the next period the capital generates an additional (expected) revenue $Z F_1^+ \beta^+$. The non-depreciated capital left in the next period also reduces the need for investment in the next period, which gives the firm expected saving equal to $C^+ \cdot (1 - \delta^+)$. Equation (6) can be interpreted in a similar way. The structure of the three equations implies that the optimal level of β is determined by equation (4) alone and is independent of the other two control variables. K^+ and L^+ are determined jointly by equations (5) and (6).

First I show the effect of financing constraint on capital accumulation. The arguments of the functions are dropped if no confusion would arise. From equations (5) and (6), we obtain

$$\frac{\partial K^+}{\partial \theta} = \frac{C_2 E\{F_{22}^+ - aF_{12}^+\beta^+\}}{Z\Delta E\{(\beta^+)^2[F_{11}^+F_{22}^+ - (F_{12}^+)^2]\}} > 0\,. \tag{7}$$

The sign follows from the regularity conditions of the production and cost functions. The above partial effect states that a period of tight finance (lower θ) would be followed by a period of low capital stock, understandably owing to the higher cost of investment. A partial effect on L^+ can be similarly derived:

$$\frac{\partial L^+}{\partial \theta} = \frac{C_2 E\{\beta^+(aF_{11}^+\beta^+ - F_{12}^+)\}}{Z\Delta E\{(\beta^+)^2[F_{11}^+F_{22}^+ - (F_{12}^+)^2]\}} > 0\,. \tag{8}$$

We next show how a variable capacity utilization rate could smooth the disturbances. From equation (4),

$$\frac{\partial \beta}{\partial \theta} = \frac{C_2\delta'}{ZF_{11}K - C\delta''} > 0\,. \tag{9}$$

This impact effect shows the role of β in the process of *intertemporal* substitution of capital. To illustrate, consider a period of tight financing. As has been shown, tight financing causes the capital stock to go down in the next period. Equation (9) then states that firms respond to this situation by *reducing* utilization rates in the current period in order to conserve capital for the next period, although the action is unlikely to totally undo the effect of the constraint. In this mechanism, variable capacity utilization smoothes the flow of capital service *intertemporally* by using capital more intensively when the replacement cost is low, and less intensively when the replacement cost is high.

The impact effect of the current capital stock on the utilization rate is

$$\frac{\partial \beta}{\partial K} = \frac{ZF_{11}\beta_t}{ZF_{11}K_t - C\delta''} < 0\,. \tag{10}$$

This result characterizes the *intratemporal* substitution of capital: firms choose a higher capacity utilization rate when the existing level of capital stock is low. That is, existing capital is used more aggressively when the available stock is low (that is, higher marginal productivity), and less aggressively when the stock is high (that is, lower marginal productivity).

Both the inter- and intratemporal substitution mechanisms help smooth the flow of capital services when the available capital stock is changing. Remarkably, all of these changes take place through adjustments in the utilization rate.

2.2 *Propagation*

Here I show that financing shocks in the current period affect output in current and future periods. Since capital and labor stocks are predetermined, it is straightforward that the current output is affected through changes in the utilization rate:

$$\frac{\partial F(\beta K, L)}{\partial \theta} = F_1 K \frac{\partial \beta}{\partial \theta} > 0\,. \tag{11}$$

Together with (9), this result also implies that output and capacity utilization move in the same direction during the period of impact. This is an essential element for the procyclical capacity utilization that will be depicted in section 3.1.

The effect on the next period is observed by taking the derivative of F^+ with respect to θ. By making use of the results of (7) and (8), we have

$$\frac{\partial F(\beta^+ K^+, L^+)}{\partial\theta} = F_1^+\left(\Gamma\frac{\partial K^+}{\partial\theta}\right) + F_2^+\frac{\partial L^+}{\partial\theta} > 0,$$

where

$$\Gamma = E\left\{\beta^+ + K^+\frac{\partial\beta^+}{\partial K^+}\right\}$$
$$= E\left\{\beta^+\left(1 - \frac{ZF_{11}^+K^+}{ZF_{11}^+K^+ - C^+(\delta^+)''}\right)\right\} > 0.$$

The last inequality sign follows because $\beta^+ > 0$, $ZF_{11}^+K^+ < 0$, and $C^+(\delta^+)'' > 0$, so that the fractional term is positive and less than one for every possible realization of the stochastic variable θ^+. This partial effect indicates that financing constraints affect the next period's output through the accumulation of capital and labor stocks. Before the stocks are restored to their original levels, the effects of financing shocks are propagated further into future periods.

It is not surprising that financing shocks affect future output through stock accumulation. What is less apparent is the fact that financing shocks also affect the current period's output even though current stocks are predetermined. This is because of the production smoothing motive which prompts firms to react before the shock's effect materializes on stocks. It means that a healthy dose of credit supply not only increases future output through stock formation, but also stimulates current output by encouraging capital consumption. Conversely, a credit contraction depresses current output because of the higher user cost of capital.

2.3 *Effects on the Labor Market*

The model also has plausible implications for labor productivity. I first show that the marginal product of labor is procyclical. By taking the result of (9) and noting that K and L are predetermined, the effect of a finance disturbance on the marginal product of labor is

$$\frac{\partial F_2(\beta K, L)}{\partial\theta} = F_{21}K\frac{\partial\beta}{\partial\theta} > 0.$$

This means that, with easy finance, the marginal product of labor increases. This is simply a result of the higher capital service to labor ratio. Together with the positive contemporaneous output effect, this implies that the marginal product of labor is procyclical.

Other stylized facts about the labor market include the average labor product being procyclical, the existence of the phenomenon of labor hoarding, and that movements of labor productivity lead employment (McCallum 1989). These stylized facts can be easily established in this model. With regard to the procyclicality of average labor product, we note that with a higher θ, production in the current period increases while employment remains unchanged (predetermined). Therefore, the output to labor ratio moves upward. The phenomenon of labor hoarding can be similarly induced: with a lower θ, output in the current period decreases while employment is not correspondingly reduced within the period. To see that changes in labor productivity lead employment, we combine the above observations with the result of equation (8). Together, they show that, while the average labor product is up in the current period, employment does not increase until the next period.

Many of the above results stem from the setup whereby the decision with regard to period t's labor employment is made before the shock θ_t is observed. Similar assumptions are used in other studies including Burnside, Eichenbaum, and Rebelo (1993). This setup is rationalized by costly hiring and firing, so that employment does not immediately adjust to shocks. A consequence of the quasi-fixed labor stock is that, with some modifications to the model, the labor stock could provide yet another source to buffer against financial shocks, as described below. First, we allow for a fractional use of labor stock by making the flow of labor services depend on the product of the stock and work efforts. Then, because of labor hoarding, firms facing financing constraints could demand higher work efforts as well as increase capacity utilization to smooth production. The dynamic effects of work efforts and labor hoarding are studied in a number of recent papers including Burnside, Eichenbaum, and Rebelo (1993) and Sbordone (1996, 1997); they are not pursued further in this paper.

3. CALIBRATION

In this section, the stochastic dynamic model is numerically calibrated. The results enable us to study the impulse responses of the choice variables, and also to examine the shock's long-run effect on capital stocks.

The Model

Specific function forms are assigned to the maximization problem (1).

$$F(\beta_t K_t, L_t) = ((\beta_t K_t)^q + L_t^q)^{\frac{1}{q}}, \quad q < 1, \tag{12}$$

$$C(I_t, \theta_t) = 1 + r + e^{\upsilon(I_t - \theta_t)}, \quad \upsilon > 0, \tag{13}$$

$$\delta(\beta_t) = \frac{1}{\omega}\beta_t^{\omega}, \quad \omega > 1, \tag{14}$$

$$\theta_{t+1} = (1-\rho)\bar{\theta} + \rho\theta_t + \eta_{t+1}; \quad \eta \sim iid\ N(0, \sigma^2). \tag{15}$$

The production function takes the form of constant elasticity of substitution, with the elasticity (τ) equals $1/(1 - q)$. Equation (13) is discussed in the next paragraph. Equation (14) follows Greenwood, Hercowitz, and Huffman (1988). Equation (15) specifies the stochastic process of the financial shock, which implies a symmetric distribution of θ_t around the long-run average value $\bar{\theta}$.

Equation (13) is the per dollar cost of borrowing I_t, so that the total cost of borrowing is $C(I_t,\theta_t)\, I_t$. r is the opportunity cost of funds and $e^{\upsilon(I_t-\theta_t)}$ is the markup of the borrowing cost over the opportunity cost. With the assumption of imperfect information, the markup can be interpreted as being associated with the borrower's information cost. Given a level of investment, the size of the markup is influenced by a stochastic variable θ_t and is also controlled by a constant parameter υ. The economic interpretation of θ_t may depends on the value of υ, and the result is a very flexible way to model financing constraint. For instance, if υ is sufficiently large, θ_t becomes the threshold of borrowing, beyond which the cost approaches infinity quickly.[2] If υ is small, the cost rises gradually, and the curve resembles a textbook style cost function. Figure 1 illustrates both cases. Alternatively, a cost function of $C(0, \theta_t)$ makes the per dollar cost invariant to the amount borrowed, and changes in θ_t shift the entire cost schedule of external finance.

The choice variables are the utilization rate β_t and the amount of capital investment i_t, and the state variables are θ_t and K_t. Although we could also make labor hiring (l_t) a choice variable and thus L_t another state variable, a system of three states is very difficult to evaluate numerically.[3] Therefore we assume that labor is fixed throughout the simulation, and thus $I_t = i_t$. This simplification also enables us to highlight the role of variable capacity utilization.

The firm then faces the following maximization problem:

$$\max_{\{\beta_t, I_t\}} : \sum_{t=1}^{\infty} \Delta^{t-1}\{Z[(\beta_t K_t)^q + L^q]^{\frac{1}{q}} - (1 + r + e^{\upsilon(I_t - \theta_t)})I_t - WL\} \tag{16}$$

2. To be precise, because the cost function is smooth, it is not possible to pinpoint a specific level for which the cost changes from a finite number to infinity. Rather, the cost changes quickly, albeit smoothly, in the neighborhood of θ_t, and the size of the neighborhood shrinks as υ increases.

3. Using the solution method described in Appendix B, a system of six nonlinear equations has to be solved numerically for six parameters in a two-states system. For a three-states system, eighteen parameters need to be solved from a system of eighteen nonlinear equations. If solvable at all, numerical solutions in such case would be imprecise at best.

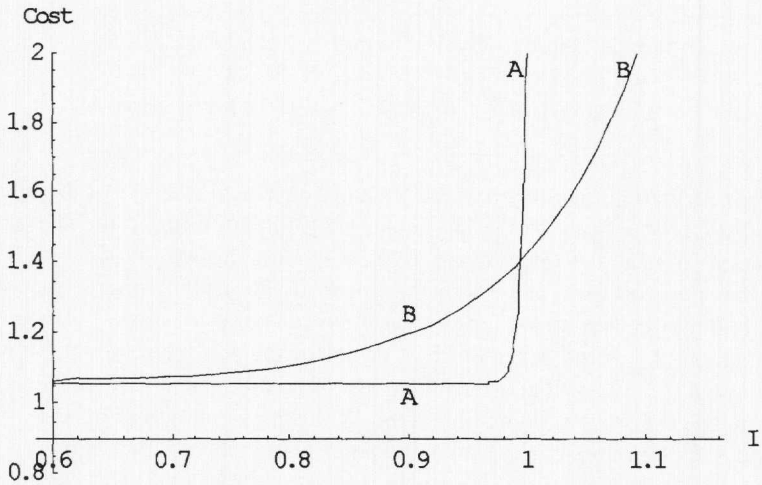


FIG. 1. Illustrations of the Cost Function (13). Curve *AA* is illustrated with $\{v, \theta_t\} = \{200, 1\}$, and curve *BB* is illustrated with $\{v, \theta_t\} = \{10, 1.1\}$.

$$st:\ K_{t+1} = \left(1 - \frac{1}{\omega}\beta_t^{\omega}\right)K_t + I_t\,,$$

$$\theta_{t+1} = (1-\rho)\bar{\theta} + \rho\theta_t + \eta_{t+1}\,. \tag{17}$$

Solving the Model

We again use a superscript "+" to denote next-period variables. The stochastic Euler equations of the choice variables β_t and I_t are as follows, respectively (see Appendix A for the derivation):

$$-[A]^{\frac{1}{q}-1}\omega^{-1}K^{q-1}\beta^{q-\omega} + \Delta E\{\Omega^{+}|\theta, K\} = 0\,, \tag{18}$$

$$-e^{\upsilon(I-\theta)}(1+\upsilon I) - (1+r) + Z\omega\Delta E\{\Omega^{+}|\theta, K\} = 0\,, \tag{19}$$

where $\Omega = A^{\frac{1}{q}-1}K^{q-1}\beta^{q-\omega}\left[1 - \left(1 - \frac{1}{\omega}\right)\left(1 - \frac{1}{\omega}\beta^{\omega}\right)\right],$

$$A = \beta^{q}K^{q} + L^{q}\,.$$

Because closed-form solutions of β and I are not available, we resort to numerical approximations. The solution technique used here is a hybrid of perturbation and projection methods (Judd 1998).

The perturbation method uses local information to approximate policy functions. In economics applications, the local information is usually derived from the deterministic steady state. Specifically, consider the feedback control laws:

$$\beta(K, \theta) = \beta^* + \beta_K^*(K - K^*) + \beta_\theta^*(\theta - \bar{\theta}) + \beta_{K\theta}^*(K - K^*)(\theta - \bar{\theta}),$$

$$I(K, \theta) = I^* + I_K^*(K - K^*) + I_\theta^*(\theta - \bar{\theta}) + I_{K\theta}^*(K - K^*)(\theta - \bar{\theta}),$$

where β^*, K^*, and $\bar{\theta}$ are steady-state values obtained from the corresponding deterministic control problem. These policy functions are Taylor series of the controls approximated around the deterministic steady state. Judd (1998) shows that the values of the unknown parameters (β_K^*, β_θ^*, $\beta_{K\theta}^*$, etc.) can be solved from the Euler equations of the control variables. Depending on the nature of the problem, values of the variables can be deduced sequentially or solved all at once from systems of equations. This latter approach is in line with the projection method also discussed by Judd (1998). Once the polynomials' coefficients are solved, values of the controls are easy to obtain for each given state (K) and shock (θ). Appendix B sketches the procedures.

Steady-State Solutions and Parameter Values

We need the steady-state values of β and I and all other parameter values to approximate policy functions.

To find out the value of β in the deterministic steady state, that is, β^*, we transform the stochastic Euler equation (18) into a nonstochastic one by converting θ to its long-run average value $\bar{\theta}$, imposing steady-state conditions (for example, $K^+ = K$, etc.), and eliminating the expectations operator. Then β^* can be derived from the equation. I^* can be obtained by recognizing that investment must equal the depreciation of capital in the steady state, and that the depreciation rate is $\frac{1}{\omega}\beta^\omega$. Therefore the solutions for β and I in the deterministic steady state are

$$\beta * = \left(\frac{\omega(1-\Delta)}{\Delta(\omega-1)}\right)^{\frac{1}{\omega}}, \tag{20}$$

$$I * = \frac{(1-\Delta)}{\Delta(\omega-1)} K * . \tag{21}$$

Next, we make both control variables functions of K and θ, and assign a conditional density function to θ^+ with domain Q. Equations (18) and (19) then become:

$$-[A]^{\frac{1}{q}-1}\omega^{-1}K^{q-1}\beta^{q-\omega} + \Delta\int_Q \Omega^+ P(\theta^+|\theta)d\theta^+ = 0, \tag{22}$$

$$-e^{\upsilon(I-\theta)}(1+\upsilon I) - (1+r) + Z\omega\Delta\int_Q \Omega^+ P(\theta^+|\theta)d\theta^+ = 0. \tag{23}$$

We now assign parameter values to the dynamic model characterized by the above two equations. We have quarterly time periods in mind when selecting the values. Given that the shadow price of capital is higher in the presence of financing constraints, the time discount factor (Δ) should be lower than otherwise, and so we set it to 0.90. The creditor's opportunity cost of funds (r) is 6 percent. The steady-state level of the interest rate on commercial loans, $r + e^{\upsilon(I^*-\bar{\theta})}$, equals 10 percent. The steady-state level of the capital depreciation rate ($\frac{1}{\omega}\beta^{*\omega}$) is 3 percent. The steady-state level of the capital stock, K^*, is normalized to one. In addition, let $\{\upsilon, \tau, \rho\} = \{200, 0.7, 0.9\}$.

Given the above exogenous parameter values, we can solve for the remaining ones with the model. The steady-state labor stock L^* is determined in such a way that, in the steady state, the elasticity of output with respect to capital is 0.5. By having equation (20) together with the depreciation rate ($\frac{1}{\omega}\beta^{*\omega}$) equal to 3 percent, we can solve for ω. This in turn gives the value of β^*. I^* equals 0.03, which is the depreciation rate times the steady-state capital stock. Given this value and with the steady-state per dollar borrowing cost function ($1 + r + e^{\upsilon(I^*-\bar{\theta})}$) equal to 1.10, we can solve $\bar{\theta}$ for a given value of υ. For the price parameter Z, the value is determined from the nonstochastic version of equation (19). Finally, the standard deviation σ has a value equal to 0.01, which is one-third of the steady-state investment level.

The polynomials of policy functions can now be estimated. Appendix B sketches the procedures.

3.1 Impulse Responses

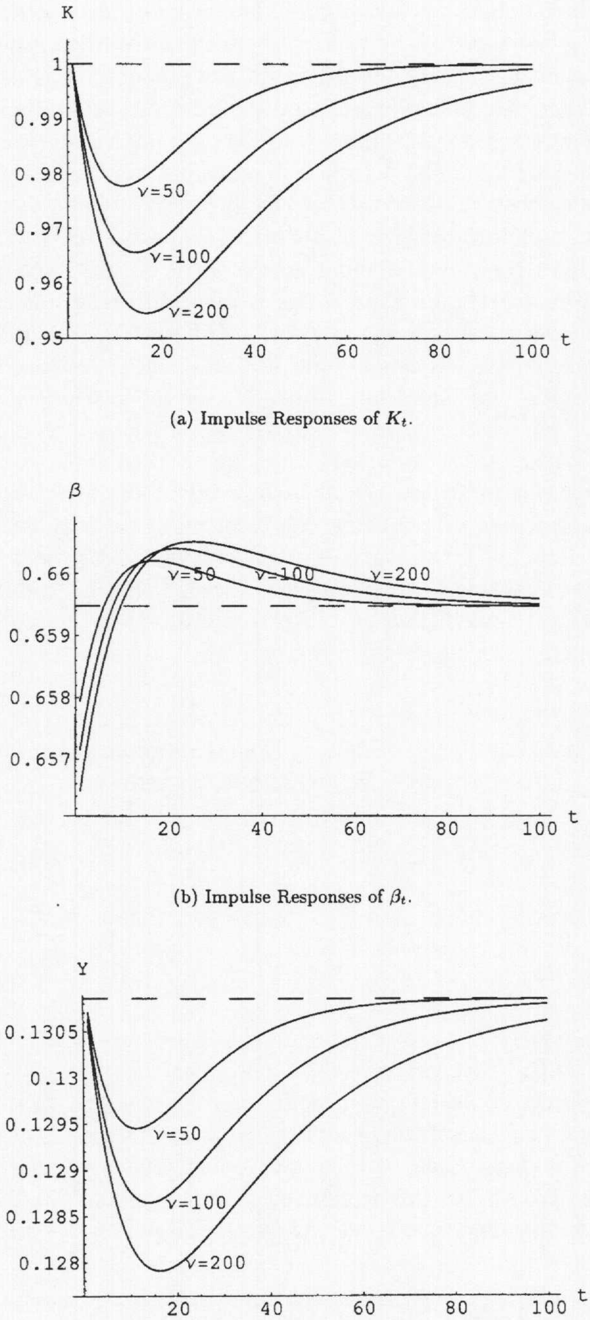
Impulse responses of K_t, β_t, and Y_t are presented and compared based on estimated policy functions from models with different values of υ. The function of K_{t+1} is obtained by substituting policy functions of β_t and I_t into (17). The initial shock takes place at $t = 1$, and the magnitude equals the negative of one standard deviation of the distribution of θ_t. A set of 100-period horizon figures is plotted in Figure 2.

As the figure shows, if the financing constraint is more stringent (a larger υ), the responses of the variables are larger and the effects of the shock last longer.

The first panel depicts the impulse responses of K_t. Because the capital stock is predetermined, the negative financing shock does not adversely affect K_t in the first period. The second panel relates to β_t which drops below the steady-state level upon impact, and climbs up gradually, overshooting the steady-state level before reverting back. The negative impact effect is predicted in equation (9) where we point out that this is the capacity utilization's intertemporal substitution of capital mechanism. However, when the adverse effect on capital sets in starting from period 2, firms respond by increasing the utilization rate in order to partially offset the drop in the flow of capital services. This is the capacity utilization's intratemporal substitution of capital mechanism, and is shown in equation (10).

The impulse response of output in the third panel has a similar shape to that of the capital stock, except that output has an immediate response to the shock owing to the change in capacity utilization. This is also shown in equation (11).

An interesting observation arising from comparing the impulse responses of β_t and Y_t is that the cyclicality of β_t seems inconclusive. On one hand, the impact of the



(a) Impulse Responses of K_t.

(b) Impulse Responses of β_t.

(c) Impulse Responses of Y_t.

FIG. 2. Impulse Responses. Dashed horizontal lines denote deterministic levels of the variables.

shock moves both variables downward, providing grounds for a procyclical β_t. On the other hand, the two variables move in the opposite direction in subsequent periods, implying a counter-cyclical β_t. While it is difficult to judge which effect, in general, is likely to dominate in a dynamic process, it can be shown that the model is capable of generating a procyclical β_t. To this end, we simulate the model to generate time series data for Y_t and the β_t for one hundred periods, and regress the generated Y_t on β_t and a constant. The coefficient for β_t is 0.373 with a t value of 3.301. Compared to the results from other models (ex. Greenwood, Hercowitz, and Huffman 1988; Kydland 1995), this correlation appears to be on the smaller side.[4] Nevertheless, the positive correlation shown here is assuring because it indicates that adding nominal financing shocks to a standard real business cycle model with real shocks will not change the cyclicality of the utilization rate in a significant way.

To demonstrate the output smoothing effect of a variable capacity utilization rate, Figure 3 plots the impulse responses of output from a model that assumes β_t is constant at the steady-state level (the short-dashed curve), and from a model that assumes β_t is flexible (the solid curve). Both assume $\upsilon = 200$. Based on the simulated data,[5] at the trough output decreases by 2.251 percent if capacity utilization is allowed to respond, and by 2.368 percent if it is held constant. On the other hand, the swing of β_t is about 0.6 percent of its steady-state level. Therefore, about 5 percent of the loss in output is avoided (*that is,* $(2.251-2.368)/2.368 \approx -0.05$) with there being only a 0.6 percent change in the utilization rate.

3.2 Long-Run Distributions of Capital

The transition probability $P_{lm,ij}$ of moving from the state characterized by $K = K_i$ and $\theta = \theta_j$ to the one represented by K_l and θ_m can be expressed as

$$\begin{aligned} P_{lm,ij} &= \text{prob}[K^+ = K_l, \theta^+ = \theta_m \mid K = K_i, \theta = \theta_j] \\ &= \text{prob}[K^+ = K_l \mid K = K_i, \theta = \theta_j] \cdot \text{prob}[\theta^+ = \theta_m \mid K = K_i, \theta = \theta_j] \\ &= \text{prob}[K^+ = K_l \mid K = K_i, \theta = \theta_j] \cdot \text{prob}[\theta^+ = \theta_m \mid \theta = \theta_j] . \qquad (24) \end{aligned}$$

The first equal sign arises because the conditional distribution of K^+, which is governed by $K^+ = h(K, \theta)$, is independent of the conditional distribution of θ^+. The second equal sign follows because the distribution of θ^+ is governed by the stochastic equation (15), which is independent of K. The first probability in (24) equals one for some l and zero for others because $K^+ = h(K, \theta)$ uniquely determines K^+, given K and θ. The second probability will be computed from equation (15). For numerical computations, we divide the state space (K) into n grid points and the shock space

4. We do not compare the result to the figure in the data, because it is difficult to find a satisfactory empirical counterpart of the utilization rate (Greenwood, Hercowitz, and Huffman 1988).

5. The minimum values of the simulated Y_t are 0.12791 and 0.12776 for models assuming a variable and constant β_t, respectively. The steady-state output is 0.13086. The minimum and maximum values of the simulated flexible β_t are 0.6565 and 0.6605, respectively, and the steady-state level is 0.6595.

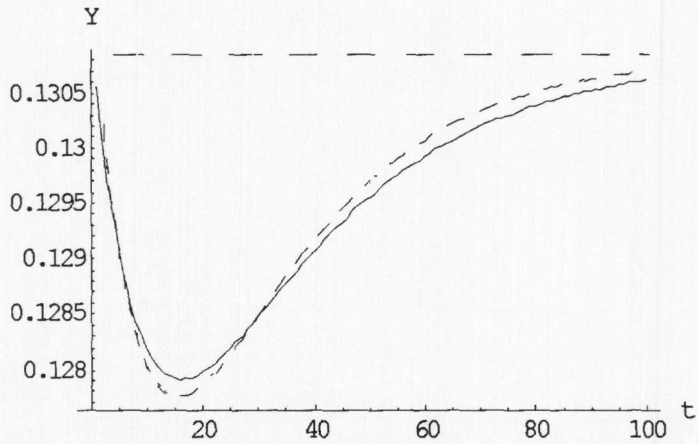


FIG. 3. Output Smoothing. The solid curve is derived from a model assuming a variable β_r, and the short-dashed curve assumes a constant β_r.

into m grid points, and we can thus form an $nm \times nm$ transition matrix P with elements $P_{lm,\,ij}$ to be calculated from (24). Assuming this model possesses a unique asymptotic distribution for K, Stokey and Lucas (1989) show that iterations on the transition matrix must converge to a unique distribution.[6] This is true for all possible initial distributions of K and θ. Figure 4 shows the long-run distribution of the capital stock.

Notice that the first moment of the distribution is virtually the same as the steady-state level. It implies that, with symmetrically distributed financing shocks, financial uncertainty does not have a long-run impact on capital accumulation. A welfare implication about credit supply is that bank loan fluctuations do not result in a first-order welfare loss in the long run, and the economy can only have a second-order welfare gain as a result of regulations and stabilization policies aimed at minimizing fluctuations in the credit sector.

4. CONCLUSION

This paper shows that firms may adjust margins to smooth production when the cost of financing fluctuates. In particular, we emphasize the role of variable capacity utilization, showing that it gives rise to both inter- and intratemporal substitutions of capital. The results from this paper show that the presence of financing constraints reinforce several business cycle phenomena, but the long-run effect on the level of

6. The long-run distribution of capital can also be obtained through simulation based on the transition function of K. The answers should be asymptotically the same.

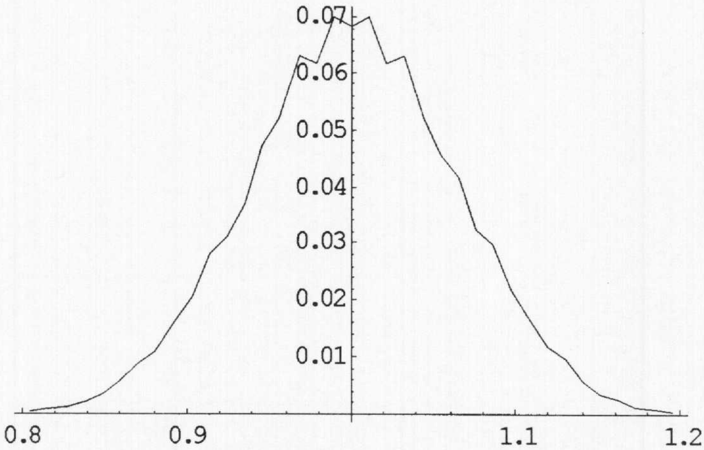


FIG. 4. The Long-Run Distribution of the Capital Stock. The mean and standard deviation of the distribution are 1.001 and 0.354, respectively.

capital stock is likely to be neutral. The latter implies that the economy only benefits a second-order welfare gain from policies aimed at stabilizing credit supplies.

It is important to point out that although the model has the ability to generate some stylized facts concerning business cycles, it does not necessarily explain all business cycle phenomena, nor does it claim that the financing constraint is the only important cause of business cycles. Instead, the author hopes that this paper has provided a useful framework for studying investment and production behavior under financing constraints, within which other business cycle factors may also be incorporated.

APPENDIX A: DERIVING EULER EQUATIONS

Following Sargent (1987), we define $u_t \equiv (1 - \frac{1}{\omega}\beta_t^{(\omega)})K_t$ as a control variable replacing β_t. We then have the Bellman equation of the problem in (16):

$$V(K_t;\theta_t) = \max_{u_t, I_t} \left\{ \begin{array}{l} Z\left[\omega^{\frac{q}{\omega}} K_t^q \left(1 - \frac{u_t}{K_t}\right)^{\frac{q}{\omega}} + L^q\right]^{\frac{1}{q}} - \left(1 + r + e^{\upsilon(I_t - \theta_t)}\right) I_t - WL \\ \qquad + \Delta E V(u_t + I_t; \theta_{t+1}) \end{array} \right\}$$

subject to (15). The first-order conditions of u and I are

$$(u) \qquad -Z[A]^{\frac{1}{q}-1}\left[\omega\left(1-\frac{u}{K}\right)\right]^{\frac{q}{\omega}-1} K^{q-1} + \Delta E\{V_1(u+I,\theta^+)|K,\theta\} = 0\,, \quad (25)$$

$$(I) \qquad -e^{\upsilon(I-\theta)}(1+\upsilon I) - (1+r) + \Delta E\{V_1(u+I,\theta^+)|K,\theta\} = 0\,, \quad (26)$$

where $A \equiv \omega^{\frac{q}{\omega}} K^q \left(1-\frac{u}{K}\right)^{\frac{q}{\omega}} + L^q\,.$

The value of V_1 in (25) and (26) can be solved by differentiating the Bellman equation with respect to K:

$$V_1(K,\theta) = Z[A^+]^{\frac{1}{q}-1}\omega^{\frac{q}{\omega}}K^{+q-1}\left(1-\frac{u^+}{K^+}\right)^{\frac{q}{\omega}-1}\left[1-\left(1-\frac{1}{\omega}\right)\frac{u^+}{K^+}\right].$$

After substituting the appropriate values of V_1 into equations (25) and (26) and noting that $K - u = \frac{1}{\omega}K\beta^{\omega}$, we obtain the stochastic Euler equations of (18) and (19).

APPENDIX B: NUMERICAL APPROXIMATIONS

We use the following polynomials to approximate the policy functions:

$$\beta(K,\theta) = \beta^* + \beta_K^*(K - K^*) + \beta_\theta^*(\theta - \bar{\theta}) + \beta_{K\theta}^*(K - K^*)(\theta - \bar{\theta})\,, \quad (27)$$

$$I(K,\theta) = I^* + I_K^*(K - K^*) + I_\theta^*(\theta - \bar{\theta}) + I_{K\theta}^*(K - K^*)(\theta - \bar{\theta})\,. \quad (28)$$

These polynomials are substituted into (22) and (23), and the integrations are numerically evaluated using Gaussian quadratures. The results are two functions of six undetermined parameters: β_K^*, β_θ^*, $\beta_{K\theta}^*$, I_K^*, I_θ^*, and $I_{K\theta}^*$. We therefore need four more equations to obtain the six parameter values. We get two of these by differentiating (22) and (23) with respect to K, and the other two by differentiating with respect to θ. We then derive the six parameter values from the system of six nonlinear equations.

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